Multidimensional-Manifold Schrödinger / SU(4)-Spin(6) Operator-Closure Reconstruction of the Free-Neutron Lifetime Baseline

Internet Archive Publication Edition v8 - Residual-Bound and Active-Plane Isotropy Audit Draft

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Bounded claim statement

This manuscript is a conditional mathematical-physics operator reconstruction. It proposes an auditable operator grammar for a candidate intrinsic neutron-lifetime baseline. It does not claim to replace the Standard Model treatment of neutron beta decay, to prove a new fundamental gauge field, or to complete a first-principles derivation of all curvature-elastic terms. The v8 revision explicitly separates the completed algebraic closure layer from the remaining proof targets: the membrane-sector action, the geometry Hamiltonian, the path-integral expansion, the spectral derivation of the curvature exponent, and the residual uncertainty budget.

Abstract

This Internet Archive v8 edition presents a multidimensional-manifold formulation of a six-internal-coordinate Schrödinger operator coupled to an SU(4)/Spin(6) sector grammar. The framework reconstructs a candidate intrinsic baseline for the free-neutron lifetime by lifting a scalar lifetime readout into a Hilbert-space state, a covariant-derivative connection, a Feshbach resonance-width description, and a finite operator closure certificate over fifteen active Pauli-sector generators. The terminology “multidimensional manifold” or “internal continuum” is used to emphasize that the construction is an effective mathematical state-space model, not a claim that ordinary spacetime has been experimentally shown to contain six additional macroscopic spatial directions.

The central numerical audit keeps the baseline value $\tau_n \approx 877.770887$ s and treats the curvature-elastic term, the spectral exponent, and higher nonlinear corrections as explicit derivation targets. The v8 revision adds a membrane-sector proof program: a membrane action S_{mem}, a geometry Hamiltonian H_{geom}, a normalized path-integral expansion, a bounded residual δ_{geom}, and physical-consistency tests such as anomaly cancellation, renormalization consistency, unitarity, and causality. The result is an Internet Archive-ready open-review manuscript with figures, formula ledger, metadata, and a bounded claim boundary.

Revision note: what was changed in v7

- Renamed the geometric language to “multidimensional manifold,” “internal continuum,” and “six-internal-coordinate manifold M6” while preserving the SU(4)/Spin(6) operator structure.
- Added a formal membrane-sector action S_{mem} as the next derivation target for the curvature-elastic correction.
- Added a geometry Hamiltonian H_{geom} coupled to the fifteen active SU(4)/Spin(6) channels.
- Added a path-integral expansion section that makes the $\alpha/45$ term a target theorem rather than a bare ansatz.
- Added an explicit residual budget δ_{geom} and a sensitivity figure.
- Added a spectral-derivation target for the delayed curvature-rise exponent.
- Added physical-consistency tests: anomaly cancellation, renormalization consistency, unitarity, and causality.
- Added Internet Archive metadata, upload notes, figure files, and manifest.

Figure 1. Operator-closure reconstruction stack

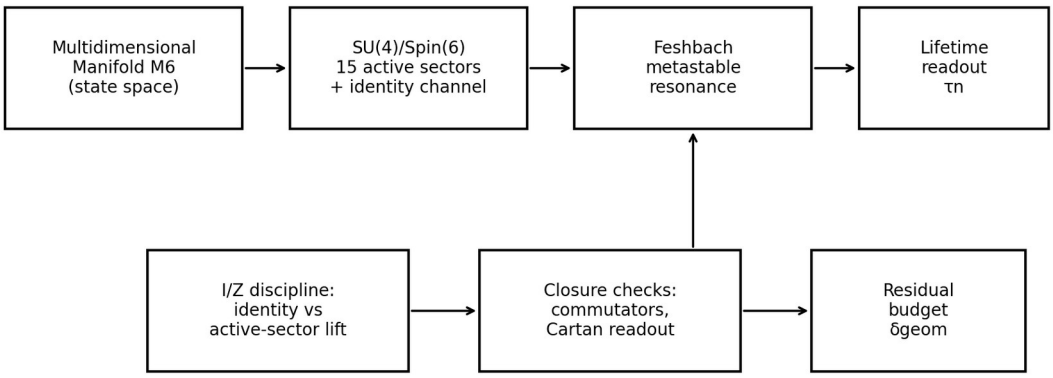

Multidimensional Manifold M6 (state space)
SU(4)/Spin(6) 15 active sectors + identity channel
Feshbach metastable resonance
Lifetime readout τn
I/Z discipline: identity vs active-sector lift
Closure checks: commutators, Cartan readout
Residual budget δgeom

Figure 1. Operator-closure reconstruction stack. The finite lifetime readout is embedded into a multidimensional-manifold state, an SU(4)/Spin(6) sector grammar, a Feshbach resonance-width layer, and a residual-bounded prediction channel.

1. Problem setting and mathematical status

The free-neutron lifetime remains an important precision observable because different experimental methods, especially beam and bottle/storage methods, have historically produced non-identical central values. The present manuscript does not attempt to replace the Standard Model calculation of neutron beta decay. Instead, it constructs an independent operator-closure baseline whose value can be compared against Standard Model corrections, experimental averages, beam-bottle discrepancies, and future high-precision data.

The basic status is conditional: the finite algebraic closure and the numerical audit are reproducible inside the stated operator grammar, while the curvature-elastic term, spectral exponent, and nonlinear residual remain explicitly identified proof targets. This claim boundary is intentional and is retained for Internet Archive release so that readers can reproduce the algebraic layer without being asked to accept a completed physical theory.

2. Multidimensional-manifold state space

Let x denote the ordinary external coordinate and let Y=(Y1,...,Y6) denote an internal coordinate on an effective multidimensional manifold M6. The complete state is represented as a tensor product of ordinary spatial, internal-geometric, and sector components:

$$\Psi \in L^2(R^3\_x) \otimes L^2(M6) \otimes C^4\_\text{sector} \qquad (1)$$

The term M6 is used as a mathematical continuum carrying the closure modes. It is not presented as an experimentally established literal extension of macroscopic space. In the operator grammar, the six-dimensional internal rotation group has fifteen independent rotation planes, matching the fifteen active traceless generators of SU(4):

$$C(6,2) = 6 \cdot 5/2 = 15 \qquad (2)$$

Figure 2. I/Z discipline and 15/16 sector projection

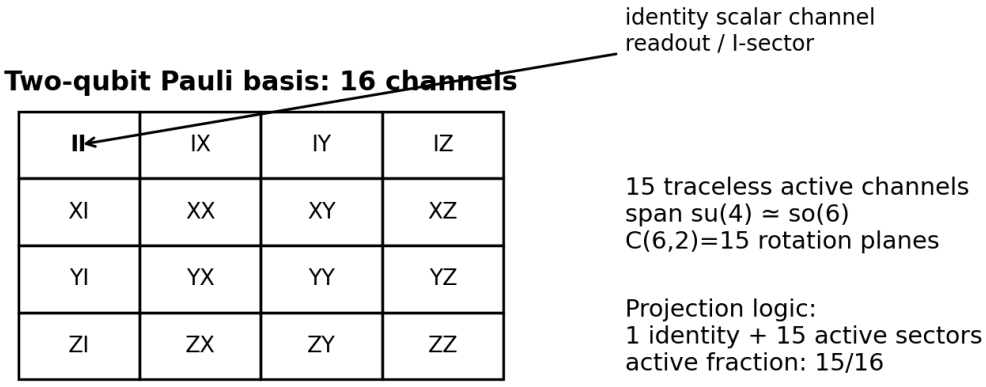


Figure 2. I/Z discipline and sector projection. The identity channel is separated from the fifteen traceless active channels. This is the algebraic origin of the 15/16 projection language.

3. I/Z discipline and operator lift

The I/Z discipline separates scalar identity readout from active-sector structure. In practical terms, an observed scalar lifetime τ_n is not used as a proof by itself. It is lifted into a reproducible invariant structure: Hilbert space, sector connection, covariant derivative, curvature commutator, and resonance-width map. The identity channel I carries ordinary scalar readout, while the active Pauli-sector channels ZI, IZ, ZZ and the remaining traceless generators carry structured closure information.

$$\Gamma_{ab} = \sigma_a \otimes \sigma_b, \quad a,b \in \{0,1,2,3\} \qquad (3)$$

$$u(4) \cong u(1) \oplus su(4), \quad su(4) \simeq so(6) \qquad (4)$$

The full two-qubit Pauli basis contains sixteen channels. The identity $\Gamma_{00}=I4$ is the scalar channel; the remaining fifteen channels form the active traceless sector. This gives the operator meaning of the "15/16 sector projection" used later in the sector action.

4. Six-internal-coordinate Schrödinger operator

The internal-continuum Schrödinger operator is written with a scalar closure potential, an active-sector potential, and a beta-exit transition Hamiltonian. In compact notation:

$$i\hbar\, D_t \Psi = [-(\hbar^2/2M6)|G|^{-1/2} D_A (|G|^{1/2} G^{AB} D_B) + V0(Y) I4 + \Phi^a(Y)\Gamma_a + H_\beta]\Psi \qquad (5)$$

$$H6 = -(\hbar^2/2M6)\Delta_{G,D} + V0\, I4 + \Phi^a \Gamma_a + H_\beta \qquad (6)$$

The ordinary electromagnetic channel is preserved as the identity-sector part of the connection, while the active-sector contribution is represented as an SU(4)-type operator channel. Setting the sector coupling to zero recovers the identity-sector minimal-coupling boundary.

$$D_A = \partial_A + i\, A_A^{cal}, \quad A_A^{cal} = (q/\hbar c) A_A\, I4 + (g_S/\hbar c) B_A^a \Gamma_a \qquad (7)$$

5. Curvature as a covariant-derivative commutator

Curvature enters as the commutator defect of the covariant derivatives. The U(1) part yields the ordinary electromagnetic field strength, while the SU(4) part produces a sector curvature. This remains a mathematical operator grammar unless and until the sector connection is physically derived and renormalized.

$$\hat{R}_{AB} = -i[D_A,D_B] = (q/\hbar c)F_{AB} I4 + (g_S/\hbar c)G_{AB}{}^a \Gamma_a \qquad (8)$$

$$G_{AB} = \partial_A B_B - \partial_B B_A + i g_S[B_A,B_B] \qquad (9)$$

6. Feshbach resonance width and beta-exit channel

The neutron is treated as a metastable closure resonance. A Feshbach projection partitions the Hilbert space into a neutron sector P_N and an exit/continuum sector $Q=I-P_N$. Eliminating the continuum yields an effective non-Hermitian Hamiltonian, whose imaginary part gives a decay width.

$$P_N = |N\rangle\langle N|, \quad Q = I - P_N \qquad (10)$$

$$H_{eff}^{(N)}(E) = H_{NN} + H_{NQ}(E - H_{QQ} + i0)^{-1}H_{QN} \qquad (11)$$

$$\Gamma_0 = -(2/\hbar)\, \mathrm{Im} \langle N|H_{NQ}(E_N - H_{QQ} + i0)^{-1}H_{QN}|N\rangle \qquad (12)$$

The sector-overlap factor is then represented as a closure amplitude multiplying the beta-exit Hamiltonian. This is a reconstruction of an intrinsic baseline, not a replacement for the Standard Model neutron-decay calculation.

$$\langle f|H_{NQ}|N\rangle = \langle f|H_\beta|N\rangle \cdot F_M, \quad |F_M|^2 = \exp(-\Sigma_M) \qquad (13)$$

7. Clock, weak gate, sector action, and main formula

The internal clock used for the baseline audit is the full neutron Compton-cycle time $h/(m_n c^2)$. The beta window is the neutron-proton-electron mass-energy difference. The weak-gate skeleton uses the dimensionless low-energy product $G_F Q_\beta^2$.

$$T_n = h/(m_n c^2) = 4.401681457647\times10^{-24}\ \mathrm{s} \qquad (14)$$

$$Q_\beta = m_n c^2 - m_p c^2 - m_e c^2 = 0.782333559310\ \mathrm{MeV} \qquad (15)$$

$$P_{weak} \approx (G_F Q_\beta^2)^2, \quad \Sigma_{weak} = -2 \ln(G_F Q_\beta^2) \qquad (16)$$

The active-sector fraction contributes 15/16; the rank-three Cartan readout contributes a $\sqrt{3}$ normalization; and the charged beta boundary carries a double electromagnetic projection α^2. The sector probability is therefore written as:

$$P_{sector} = (16\sqrt{3}/15)\,\alpha^2, \quad \Sigma_{sector} = \ln[15/(16\sqrt{3}\,\alpha^2)] \qquad (17)$$

Combining the clock, weak gate, sector action, curvature correction, and residual factor gives the conditional baseline formula:

$$\tau_n = T_n (G_F Q_\beta^2)^{-2} [15/(16\sqrt{3}\,\alpha^2)] \exp(-\alpha/45)\,(1+\delta_{geom}) \qquad (18)$$

8. Curvature-elastic correction: status and residual

The v5 formula used a first-order curvature-elastic correction normalized as one lepton curvature circulation distributed over fifteen active Spin(6) planes and three Cartan readout axes. In v7, this term is retained as the correct current reconstruction layer, but its status is clarified: it is a proof target rather than a completed theorem.

$$a_{lep} = \alpha/(2\pi) + O(\alpha^2), \quad I_{curv} = \oint a_{lep}\, d\varphi = \alpha \qquad (19)$$

$$\Delta\Sigma_{curv}^{(1)} = -I_{curv}/(15\cdot 3) = -\alpha/45 \qquad (20)$$

$$\delta\tau/\tau = \delta\Sigma + O(\delta\Sigma^2), \quad \Delta\tau \approx \tau\cdot\alpha/45 \approx 0.142\ \text{s} \qquad (21)$$

Figure 4. Residual sensitivity budget around τ_0 = 877.770887 s

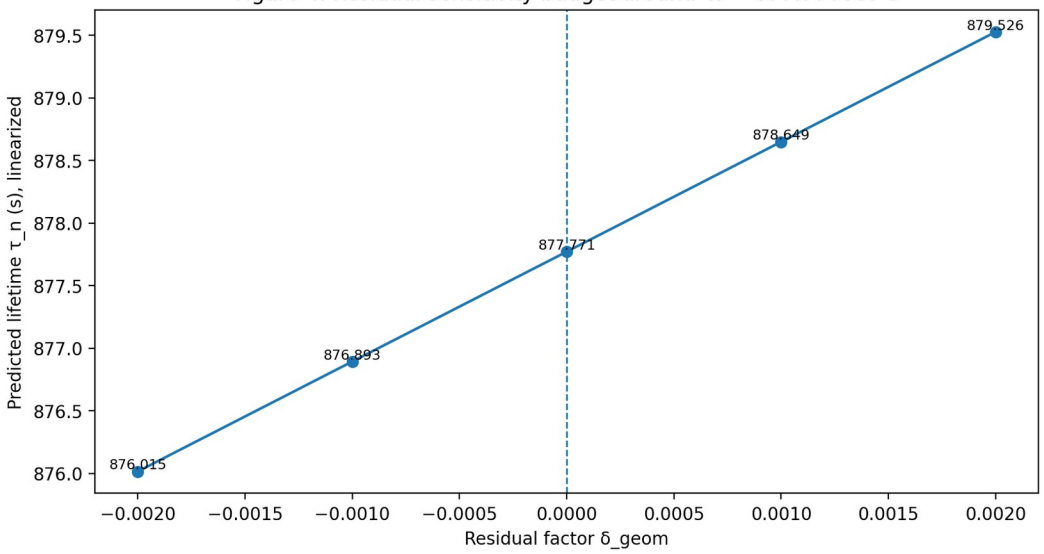


Figure 4. Residual sensitivity budget. The residual factor δ_geom is the mathematically honest location of uncomputed higher-order membrane, spectral, and renormalization corrections.

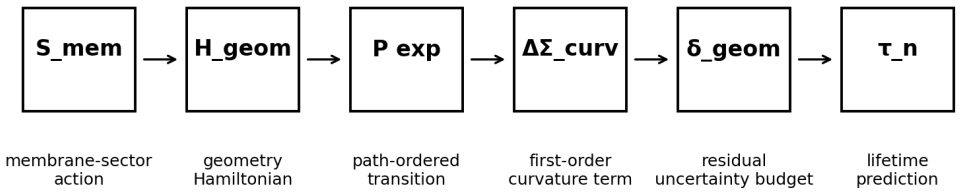
9. Proof Target A: membrane-sector action

To move the curvature-elastic ansatz toward a derived theorem, the internal manifold is treated as a quantized membrane/continuum sector with displacement w(ξ,t). The next mathematical target is to define a membrane action containing kinetic, tension, bending, and nonlinear elastic terms:

$$S_{mem} = \int dt\, d^2\xi\, [\, (\rho_m/2)\, w_{dot}^2 - (\sigma/2)|\nabla w|^2 - (D/2)(\Delta w)^2 - (\lambda 3/3)w^3 - (\lambda 4/4)w^4\,] \qquad (22)$$

This action supplies the weight for the path integral. The variables ρ_m, σ, D, λ3, and λ4 must be interpreted as effective continuum parameters of the internal manifold, not as independently measured material constants. The program is to show that the observed first-order curvature correction is a stationary or perturbative consequence of this action.

Figure 3. Proof-target chain: from continuum action to residual-bounded prediction



The v6 revision treats the curvature-elastic term as a derivation target, not a completed theorem.

Figure 3. Membrane-sector proof chain. The v8 edition explicitly separates the action, Hamiltonian, path expansion, residual budget, and final lifetime prediction.

10. Proof Target A continued: geometry Hamiltonian and path expansion

The operator for the internal geometry must couple the continuum displacement to the fifteen active SU(4)/Spin(6) channels. A compact target Hamiltonian is:

$$H_{geom} = E_{bend} + E_{nl} + \hbar\Omega_c \sum_{a=1}^{15} h_a(\hat{w},\Delta\hat{w})\,\Gamma_a \qquad (23)$$

The normalized exit amplitude can then be written as a path-integral ratio. The first-order term should recover the α/(15·3) distribution if the membrane dynamics naturally distributes one lepton curvature circulation across the fifteen internal rotation planes and three Cartan axes:

$$\Delta\Sigma_{curv}^{(1)} = -\ln\left\{ \int Dw\, \exp[iS_{mem}[w]/\hbar]\langle\chi_{exit}|Pexp[-(i/\hbar)\int H_{geom}[w]dt]|\chi_N\rangle \Big/ \int Dw\, \exp[iS_{mem}[w]/\hbar]\langle\chi_{exit}|\chi_N\rangle \right\} \qquad (24)$$

$$\text{Target theorem:}\quad \Delta\Sigma_{curv}^{(1)} = -\alpha/(15\cdot 3) = -\alpha/45 \qquad (25)$$

All terms beyond first order, including $O(\alpha^2)$, nonlinear shifts λ3 and λ4, spectral corrections, and renormalization refinements, are assigned to the residual factor δ_geom until they are derived and bounded.

11. Proof Target B: spectral derivation of the curvature exponent

The v5 framework used a diagnostic curvature exponent to describe the strongly delayed rise from the nearly flat lepton branch to the compact nonlinear baryon branch. In v7 this exponent is treated as a spectral derivation target. The required task is to expand the continuum displacement into modes and to show that the spectral density naturally generates the delayed rise:

$$w(\xi,t) = \sum_k q_k(t)\,\varphi_k(\xi), \qquad \rho_{spec}(\omega) = \sum_k \delta(\omega-\omega_k)|\langle\varphi_k|O_{curv}|\chi_N\rangle|^2 \qquad (26)$$

$$\text{Curvature readout:}\quad C(m) = C_0 + \int d\omega\, \rho_{spec}(\omega)\, K(\omega,m) \qquad (27)$$

If the spectral density of the internal continuum modes dynamically produces the delayed curvature rise, then the curvature-elastic framework becomes substantially stronger: the exponent is no longer a fitted diagnostic but a consequence of the mode spectrum.

12. Proof Target C: physical consistency tests

To satisfy the standards of mathematical physics, the operator grammar must be tested for physical consistency beyond numerical agreement. The v8 edition therefore adds the following explicit tests:

- Anomaly cancellation or a clear statement that the sector connection is an effective operator grammar rather than a fundamental gauge field.
- Renormalization consistency of the dynamical connection B_A^a and of the continuum parameters ρ_m, σ, D, λ_3, and λ_4.
- Unitarity and causality of the transition amplitude before the Feshbach projection produces the effective non-Hermitian resonance width.
- Stability of the residual budget δ_{geom} under higher-order corrections and spectral truncation.
- Recovery of the identity-sector Standard Model comparison boundary when the active-sector coupling is switched off.

13. Numerical audit and uncertainty budget

Quantity	Working value / status	Role	Claim boundary
τ_0	877.770887 s	baseline audit	conditional reconstruction value
$\alpha/45$	1.62163390×10^{-4}	first-order curvature action	derivation target via S_{mem}
δ_{geom}	not fixed	residual budget	must be bounded by expansion
15/16	active-sector projection	SU(4)/Spin(6) closure	algebraic, not alone a physical proof
$\sqrt{3}$	Cartan readout normalization	rank-three diagonal channel	operator-normalization layer
$G_F Q_\beta^2$	weak-gate skeleton	dominant suppression scale	not full SM neutron-decay calculation

The sensitivity principle is $\delta\tau/\tau \approx \delta\Sigma$ for small action shifts. Consequently, a residual δ_{geom} of order 10^{-3} corresponds to a sub-second but experimentally relevant movement of the predicted lifetime. The residual is therefore not a cosmetic parameter; it is the location where the open derivation must become quantitative.

14. Experimental and computational falsifiability

The framework becomes scientifically useful only if it can be constrained or falsified. The v8 edition retains the experimental protocol matrix and emphasizes the following discriminators:

- Straight-tube neutron experiments should exhibit a velocity-dependent correction if the continuum-geometry closure couples to transport time.

- Storage experiments may require a Lindblad-type mixing correction if boundary interactions perturb the closure modes.
- Commutator closure and the 225 ordered-pair / 120 nonzero-commutator audit must remain reproducible.
- The residual δ_geom must be bounded; otherwise the numerical agreement cannot be promoted from audit to prediction.
- The Standard Model comparison boundary must remain explicit: the operator reconstruction is a supplemental intrinsic-baseline model, not a replacement for the charged-current weak decay calculation.

15. Internet Archive upload notes

This v8 edition is prepared as an Internet Archive / Texts upload package. Recommended metadata are provided in a separate TXT file. The document is an open-review mathematical-physics preprint and should be uploaded with a title, creators, description, subject tags, license, and version note. The recommended identifier is:

alexandar-balevsky-peter-yovchev-multidimensional-manifold-neutron-lifetime-v8 (28)

The upload package includes the editable DOCX, the ready-to-read PDF, the metadata TXT, README, package manifest, and separate PNG figure files. The recommended license is CC BY 4.0 unless the authors choose a different archival license during upload.

Appendix A. Formula ledger

ID	Formula / statement
A1	$\Psi \in L^2(R^3_x) \otimes L^2(M6) \otimes C^4_{sector}$
A2	$C(6,2)=15$
A3	$u(4) \cong u(1) \oplus su(4)$, $su(4) \simeq so(6)$
A4	$i\hbar D_t\Psi = H6\Psi$
A5	$\hat{R}_{AB} = -i[D_A,D_B]$
A6	$\Gamma 0 = -(2/\hbar)\mathrm{Im}<N\|H_{NQ}(E_N-H_{QQ}+i0)^{(-1)}H_{QN}\|N>$
A7	$\tau_n = T_n(G_F Q_\beta^2)^{(-2)}[15/(16\sqrt{3}\alpha^2)]\exp(-\alpha/45)(1+\delta_{geom})$
A8	$S_{mem} = \int dt\, d^2\xi[(\rho_m/2)w_{dot}^2-(\sigma/2)\|\nabla w\|^2-(D/2)(\Delta w)^2-(\lambda 3/3)w^3-(\lambda 4/4)w^4]$
A9	$H_{geom} = E_{bend} + E_{nl} + \hbar\Omega_c \Sigma_a h_a \Gamma_a$
A10	Target theorem: $\Delta\Sigma_{curv}^{(1)}=-\alpha/45$

Appendix B. References

[1] Particle Data Group, Review of Particle Physics, Progress of Theoretical and Experimental Physics 2022, 083C01 (2022).

[2] F. E. Wietfeldt and G. L. Greene, The neutron lifetime, Reviews of Modern Physics 83, 1173-1192 (2011).

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[9] E. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, CODATA recommended values of the fundamental physical constants: 2018, Reviews of Modern Physics 93, 025010 (2021).

[10] A. Balevsky and P. Yovchev, prior OUXSPACE / TERRA SPACE preprint drafts and supplementary operator-audit packages.

[11] Recent neutron-lifetime reviews and beam-bottle comparison reports cited in the clean v5 package.

16. v7 development: completing Proof Target A as a derivation program

Purpose of this v7 expansion. The strongest current recommendation is to promote Proof Target A from a stated target into a stepwise derivation program. The central question is whether the curvature-elastic factor exp(-alpha/45) can be derived from the membrane-sector path integral rather than inserted as a reconstruction term. This section preserves the bounded claim boundary: the theorem is not declared completed, but its proof obligations are made explicit enough for expert audit.

Claim boundary retained in v7
The completed layer is the operator grammar and algebraic closure. The new layer is a proof program for the first-order curvature term. The manuscript should not state that -alpha/45 is fully derived until the lemmas below are proved and the residual delta_geom is bounded.

16.1 Target theorem A: first-cumulant curvature distribution

The proof target can be stated as a conditional theorem. Let S_mem define the internal-continuum measure, let H_geom couple the membrane displacement to the fifteen active SU(4)/Spin(6) channels, and let Pi_C denote the rank-three Cartan readout projector. If the stationary membrane measure is active-plane isotropic and Cartan-balanced, then the first cumulant of the normalized exit amplitude should satisfy:

$$\text{DeltaSigma\_curv}^{(1)} = -\alpha/(15 \times 3) = -\alpha/45$$

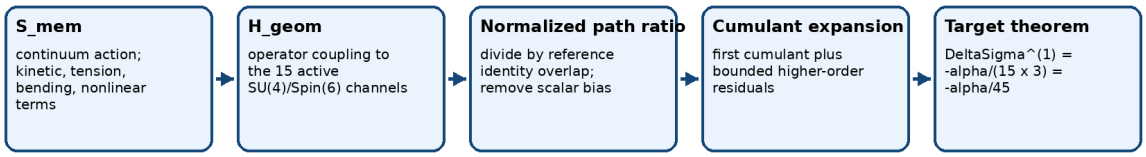
Equivalently, the curvature part of the lifetime factor becomes:

$$F_{curv} = \exp(\text{DeltaSigma\_curv}^{(1)}) = \exp(-\alpha/45)$$

This statement has three explicit components:

- a boundary curvature circulation I_curv = alpha + O(alpha^2);
- an active-plane average over fifteen Spin(6) rotation planes, giving a factor 1/15;
- a rank-three Cartan readout average, giving a factor 1/3.

Proof Target A: from membrane action to derived curvature term


S_mem
continuum action; kinetic, tension, bending, nonlinear terms
H_geom
operator coupling to the 15 active SU(4)/Spin(6) channels
Normalized path ratio
divide by reference identity overlap; remove scalar bias
Cumulant expansion
first cumulant plus bounded higher-order residuals
Target theorem
DeltaSigma^(1) = -alpha/(15 x 3) = -alpha/45

Distribution principle: one active-plane average over 15 channels, multiplied by one Cartan readout average over 3 axes.
Residual principle: all O(alpha^2), lambda3/lambda4, spectral, and renormalization terms remain in delta_geom until bounded.

Upgrade criterion:
If the first cumulant is derived and the residual is bounded, the lifetime formula moves from conditional reconstruction to residual-bounded prediction.

Figure 5. Proof Target A cumulant ladder. The v7 expansion separates the action, Hamiltonian, normalized path ratio, cumulant expansion, target theorem, and residual accounting.

16.2 Lemma structure required to complete the proof

Lemma A1 - well-posed membrane measure: The quadratic part of S_mem must define a stable Gaussian reference measure after gauge and zero-mode treatment. The nonlinear lambda3 and lambda4 terms are then handled perturbatively or through a controlled effective action.

Lemma A2 - active-plane isotropy: The stationary internal-continuum measure must distribute the first curvature insertion equally across the fifteen active SU(4)/Spin(6) channels. This gives <Pi_a> = 1/15 for each active channel.

Lemma A3 - Cartan readout normalization: The observable lifetime readout must project through the three diagonal Cartan directions without selecting one axis by hand. This gives <Pi_C,j> = 1/3.

Lemma A4 - boundary curvature circulation: The lepton boundary insertion must produce I_curv = integral a_lep dphi = alpha + O(alpha^2), where the O(alpha^2) part is not discarded but transferred to the residual budget.

Lemma A5 - first-cumulant dominance: The logarithm of the normalized path-integral ratio must admit a cumulant expansion whose first cumulant gives -alpha/45 and whose higher cumulants are bounded.

16.3 Normalized path ratio and cumulant expansion

The v7 proof program should write the exit amplitude as a normalized ratio, so that scalar normalization artifacts do not masquerade as physical curvature. Define:

$$Z_{exit} = \int Dw \exp(i S_{mem}[w]/\hbar) \langle \chi_{exit}| P \exp[-(i/\hbar) \int H_{geom}[w] dt] |\chi_N\rangle$$

$$Z_0 = \int Dw \exp(i S_{mem}[w]/\hbar) \langle \chi_{exit}|\chi_N\rangle$$

$$\Delta\Sigma_{curv} = -\log(Z_{exit}/Z_0)$$

Expanding the logarithm gives a cumulant series:

$$\Delta\Sigma_{curv} = \Delta\Sigma^{(1)} + \Delta\Sigma^{(2)} + \Delta\Sigma_{nl} + \Delta\Sigma_{spec} + \Delta\Sigma_{ren} + \ldots$$

The first-order part is the only term allowed to support the current curvature-elastic correction. Everything else must be placed in the residual until bounded:

$$\delta_{geom} = \exp(\Delta\Sigma^{(2)} + \Delta\Sigma_{nl} + \Delta\Sigma_{spec} + \Delta\Sigma_{ren} + \ldots) - 1$$

16.4 Residual bound required for journal-level strength

A high-tier journal will not require that every higher-order term vanish, but it will require a quantitative bound. A minimal acceptable target is:

$$|\delta_{geom}| \leq \varepsilon_{geom}$$

with epsilon_geom stated before comparison with the lifetime value. In practical audit language, this means the manuscript should report a table such as:

Term	Mathematical source	Current v7 status	Needed bound
DeltaSigma^(1)	first cumulant	target theorem	must equal -alpha/45

DeltaSigma^(2)	quadratic cumulant	residual	|DeltaSigma^(2)| <= eps2
DeltaSigma_nl	lambda3, lambda4 terms	residual	perturbative/stability bound
DeltaSigma_spec	mode spectrum	Proof Target B link	spectral truncation bound
DeltaSigma_ren	renormalization of B_A^a and parameters	physical consistency target	scale-stability bound

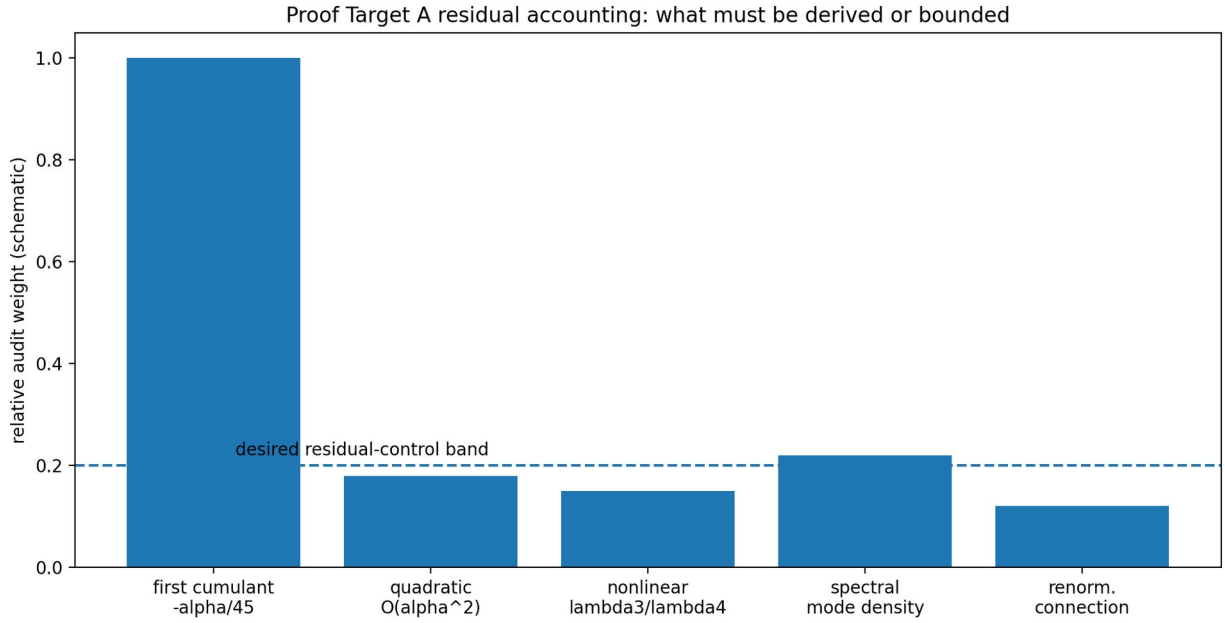


Figure 6. Residual accounting for Proof Target A. Only the first cumulant supports the current -alpha/45 correction; higher-order, nonlinear, spectral, and renormalization terms remain in delta_geom until bounded.

16.5 Concrete completion checklist for Proof Target A

Step	Deliverable	Pass criterion
A.1	Define functional domain, boundary conditions, and zero-mode treatment for w(xi,t).	S_mem has a stable quadratic sector and a controlled nonlinear sector.
A.2	Derive H_geom from the membrane variables and the fifteen active generators.	No generator leaves the SU(4)/Spin(6) closure span.
A.3	Prove active-plane isotropy or explicitly state the weaker symmetry assumption.	The factor 1/15 is not hand-inserted.
A.4	Prove Cartan readout averaging over the three diagonal axes.	The factor 1/3 is an operator-normalization result.
A.5	Evaluate the first cumulant of -log(Z_exit/Z_0).	DeltaSigma^(1) = -alpha/45.
A.6	Bound all remaining terms in delta_geom.	Numerical agreement becomes a residual-bounded prediction.
A.7	State falsifiable signatures and comparison boundary to the Standard Model.	The model remains testable and non-overclaiming.

16.6 Recommended wording for the next manuscript version

Recommended upgrade language: The v7 manuscript should say that Proof Target A has been reduced to five explicit lemmas and one residual bound. It should not say that the path-integral derivation is complete until the first cumulant and the residual estimate are actually proved. Once

those steps are completed, the central lifetime formula can be promoted from a conditional operator reconstruction to a residual-bounded derived prediction.

17. v8 addendum: numerical residual grid and active-plane isotropy tests

Purpose of this v8 addendum. The v7 proof program reduced Proof Target A to a path-integral first-cumulant theorem and a residual bound. The v8 addendum makes two additional pieces operational: a numerical residual sensitivity grid for delta_geom and a concrete active-plane isotropy test program. The manuscript still retains the bounded claim boundary: the residual is not yet derived, and active-plane isotropy remains a proof-and-falsification target rather than an asserted physical fact.

v8 claim boundary
The v8 additions improve auditability and publication readiness. They do not claim that delta_geom is already bounded from first principles, nor that active-plane isotropy is experimentally established. They define what must be proved or falsified next.

17.1 Residual sensitivity grid for delta_geom

The current audit baseline uses delta_geom = 0 to isolate the intrinsic reconstruction. Because the lifetime factor is proportional to (1 + delta_geom), the leading residual shift is:

$$\tau_n(\delta_{geom}) = \tau_0 (1 + \delta_{geom})$$

$$\Delta\tau = \tau_0 \delta_{geom}, \quad \Delta\tau / \tau_0 = \delta_{geom} + O(\delta_{geom}^2)$$

Using $\tau_0 = 877.770887$ s, the following grid quantifies the effect of representative residual values. This grid is intended as a publication audit table, not as a fitted final uncertainty.

Residual delta_geom	Resulting lifetime tau_n (s)	Shift from baseline (s)
-1.0e-03	876.893116	-0.877771
-5.0e-04	877.332002	-0.438885
-1.0e-04	877.683110	-0.087777
+1.0e-04	877.858664	+0.087777
+5.0e-04	878.209772	+0.438885
+1.0e-03	878.648658	+0.877771

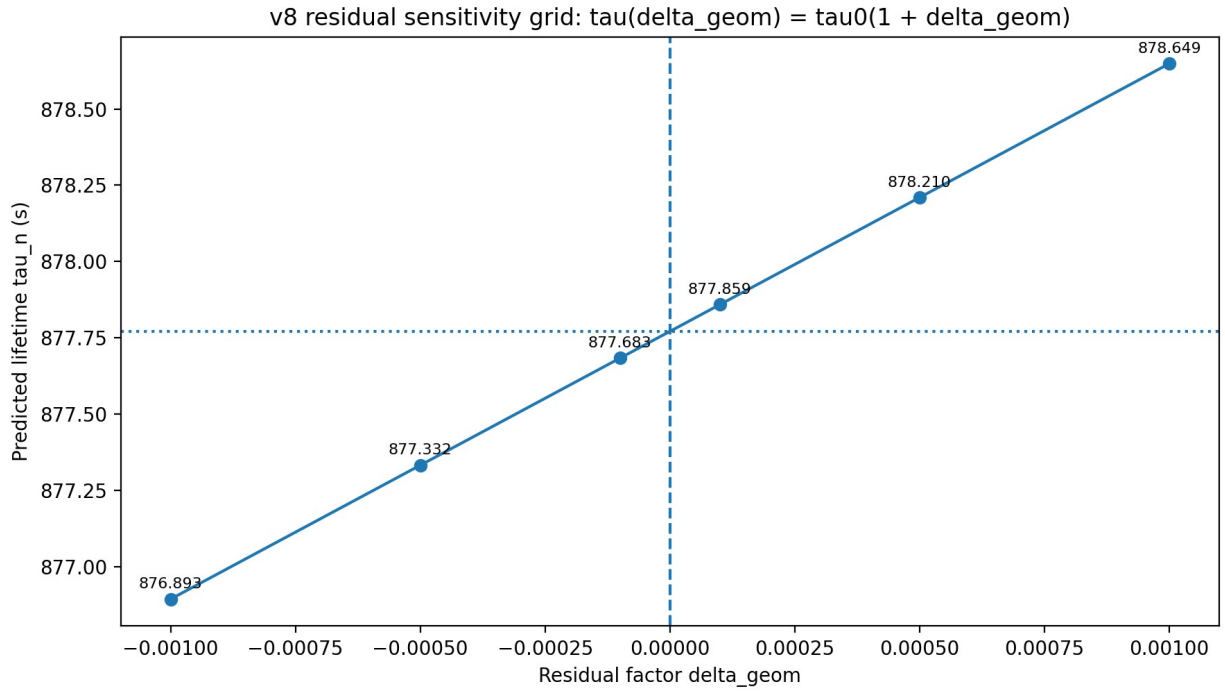


Figure 7. v8 numerical residual sensitivity grid. A residual of order 10^-3 moves the predicted lifetime by about 0.878 s, which is experimentally relevant at the beam-bottle scale.

Interpretation. A residual of order 10^-3 is not a harmless cosmetic term: it produces a sub-second motion of the predicted baseline. A journal-level version should therefore report a quantitative target of the form |delta_geom| <= epsilon_geom, with epsilon_geom chosen before comparison against experimental data.

Target class	Illustrative bound	Meaning
Archival/open-review audit	|delta_geom| <= 1e-3	sub-second sensitivity is visible and explicitly acknowledged
Stronger preprint target	|delta_geom| <= 5e-4	residual contribution kept below about 0.44 s
Predictive target	|delta_geom| <= 1e-4	residual contribution kept below about 0.088 s
High-tier proof target	controlled by O(alpha^2) and nonlinear/spectral bounds	residual is derived from the path expansion instead of assigned

17.2 Active-plane isotropy as a proof-and-falsification target

The alpha/45 term requires more than the algebraic fact that there are fifteen active SU(4)/Spin(6) directions. It requires the first curvature insertion to be distributed isotropically over the active-plane sector and then read out through the rank-three Cartan channel. In v8 this is made explicit as an audit target.

$$\langle P_{target} \rangle_{iso} = 1/(15 \times 3) = 1/45$$

$$\Delta\Sigma_{curv}^{(1)} = -\alpha \langle P_{target} \rangle_{iso} = -\alpha/45$$

$$\langle P_{target} \rangle = 1/45 + \varepsilon_{iso} \Rightarrow \Delta\Sigma_{iso} = -\alpha\, \varepsilon_{iso}$$

Any nonzero epsilon_iso is not ignored. It is transferred to the residual budget until it is bounded or experimentally excluded.

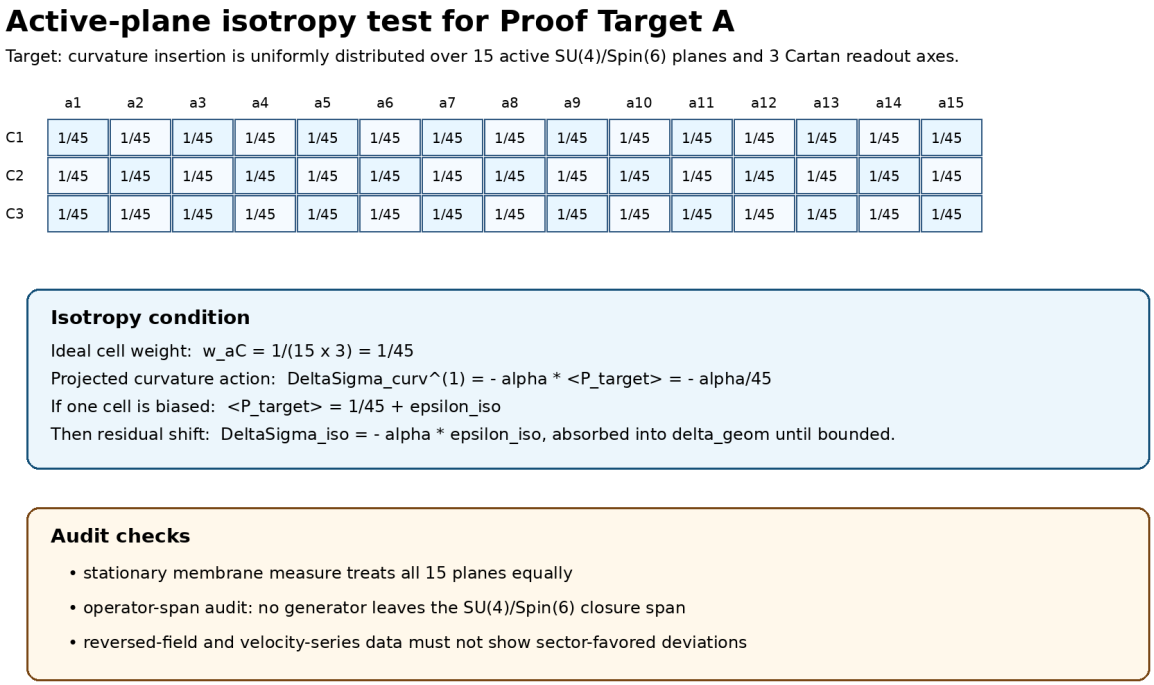

Active-plane isotropy test for Proof Target A
Target: curvature insertion is uniformly distributed over 15 active SU(4)/Spin(6) planes and 3 Cartan readout axes.
a1 a2 a3 a4 a5 a6 a7 a8 a9 a10 a11 a12 a13 a14 a15
C1 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45
C2 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45
C3 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45 1/45
Isotropy condition
Ideal cell weight: w_aC = 1/(15 x 3) = 1/45
Projected curvature action: DeltaSigma_curv^(1) = - alpha * <P_target> = - alpha/45
If one cell is biased: <P_target> = 1/45 + epsilon_iso
Then residual shift: DeltaSigma_iso = - alpha * epsilon_iso, absorbed into delta_geom until bounded.
Audit checks
• stationary membrane measure treats all 15 planes equally
• operator-span audit: no generator leaves the SU(4)/Spin(6) closure span
• reversed-field and velocity-series data must not show sector-favored deviations

Figure 8. Active-plane isotropy audit. The ideal target is a uniform 1/45 projected curvature weight across fifteen active planes and three Cartan readout axes. Deviations become epsilon_iso and enter delta_geom.

Test item	Mathematical condition	Failure mode
Stationary measure isotropy	all fifteen active planes have identical first insertion weight	one sector is preferred, shifting epsilon_iso
Operator-span audit	commutators remain inside the fifteen-channel closure span	dynamics leaks outside the declared operator grammar
Cartan readout balance	three diagonal readout axes are not arbitrarily selected	the 1/3 factor becomes a convention rather than a derived normalization
Residual correlation	unexplained shifts do not track boundary or field variables	delta_geom is hiding a physical anisotropy

17.3 Experimental discriminators for active-plane isotropy

The isotropy lemma is mathematical, but it also implies experimental discriminators. If the internal-continuum sector is isotropic, transport corrections should be controlled and reproducible rather than sector-favored. The v8 draft therefore records three practical discriminators.

- **Reversed-field polarity:** If active-plane isotropy holds, reversing magnetic field polarity should not introduce uncontrolled sector-specific sign changes in the transport correction.
- **Straight-tube velocity series:** A velocity-dependent correction should fit a stable form such as Gamma_obs(v) = Gamma_0 + A_t/v^2. Instability of A_t across controlled settings would weaken the isotropy assumption.
- **Low-loss storage extrapolation:** Different storage apparatuses, materials, and boundary configurations should extrapolate toward a common intrinsic baseline if the residual is not apparatus-driven.
- **Residual correlation audit:** Independently varied boundary or field variables should not correlate systematically with the inferred delta_geom unless the model explicitly accounts for that correlation.

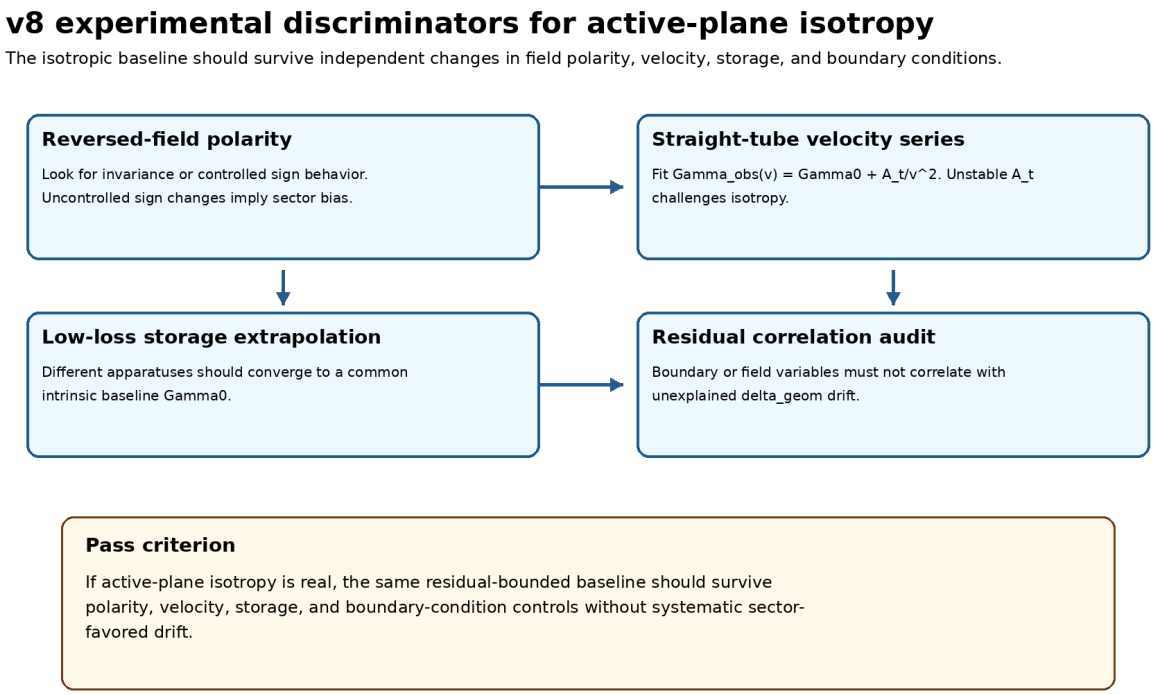

v8 experimental discriminators for active-plane isotropy
The isotropic baseline should survive independent changes in field polarity, velocity, storage, and boundary conditions.
Reversed-field polarity
Look for invariance or controlled sign behavior. Uncontrolled sign changes imply sector bias.
Straight-tube velocity series
Fit Gamma_obs(v) = Gamma0 + A_t/v^2. Unstable A_t challenges isotropy.
Low-loss storage extrapolation
Different apparatuses should converge to a common intrinsic baseline Gamma0.
Residual correlation audit
Boundary or field variables must not correlate with unexplained delta_geom drift.
Pass criterion
If active-plane isotropy is real, the same residual-bounded baseline should survive polarity, velocity, storage, and boundary-condition controls without systematic sector-favored drift.

Figure 9. Experimental discriminators for the active-plane isotropy hypothesis. The isotropic baseline must survive polarity, velocity, storage, and residual-correlation controls.

17.4 v8 upgrade language for Internet Archive

Recommended manuscript wording. The v8 edition should be described as an open-review mathematical-physics draft that has moved Proof Target A from a qualitative recommendation to a quantitative audit program. The first-cumulant theorem remains open, but the proof obligations are now explicit: derive the 1/15 active-plane factor, derive the 1/3 Cartan readout factor, compute the normalized path-ratio first cumulant, and bound delta_geom. This is the correct claim level for Internet Archive or Zenodo release.

17.5 Additional formula ledger entries introduced in v8

ID	Formula / statement
V8-1	tau_n(delta_geom) = tau_0(1 + delta_geom)
V8-2	Delta tau = tau_0 delta_geom
V8-3	<P_target>_iso = 1/(15 x 3) = 1/45
V8-4	<P_target> = 1/45 + epsilon_iso
V8-5	DeltaSigma_iso = -alpha epsilon_iso, assigned to delta_geom until bounded
V8-6	Suggested experimental fit: Gamma_obs(v) = Gamma_0 + A_t/v^2